FST 6-3 Notes
Topic: Counting Strings with Replacement
GOAL
Apply the Multiplication Counting Principle to intuit the formulas for the number of strings with replacement of length k and a set S with n elements.

SPUR Objectives

B Find the number of strings with replacement.

E Determine whether events are independent or dependent.

I Calculate probabilities in real situations.

J Use the Multiplication Counting Principle and the Strings with Replacement Theorem to find the number of ways of arranging objects.

Vocabulary

string
length of a string
independents events
dependent events

Write down all the three-letter combinations that begin with D using the letters:
A, B, C, ~~D~~, and E only once. can't repeat

DAE, DEC, DAB, DBC, DCA, DAC, DEA, DBA, DCE, DBE, DCB, DEB

$\underset{1st}{\underline{1}} \cdot \underset{2nd}{\underline{4}} \cdot \underset{3rd}{\underline{3}} = 12$ combinations

How many three-letter combinations are there using the letters: A, B, C, D, and E only once?

$\underline{5} \cdot \underline{4} \cdot \underline{3} = 60$ combinations

Multiplication Counting Principle Review

You have 4 different colored socks, 5 different colored pants, and 3 different colored shirts. How many combinations can you make?

$\underset{\text{socks}}{\underline{4}} \cdot \underset{\text{pants}}{\underline{5}} \cdot \underset{\text{shirts}}{\underline{3}} = 60$ outfits

Multiplication Counting Principle

Let A and B be any finite sets. The number of ways to choose one element from A and then one element from B is $N(A) \cdot N(B)$.

Example 1: A lottery game played in some states involves picking 3 digits from 0 to 9 in order. Describe a sample space for this experiment and determine the number of elements in the sample space.

Sample space : 000 — 999

\# of elements in sample space $\underline{10} \cdot \underline{10} \cdot \underline{10} = \boxed{1000}$

When symbols (numbers, objects, etc…) in a problem must be ordered, the ordered list is a **string.**

The number of symbols in the sting is its **length**.

string: do not think of cord or wire, instead think of a "string of pearls" where a group of objects are arranged one after the next

Example 2: Three of the questions on a science test are multiple choice with four choices each.

a) How many ways are there to answer these questions?

$$\underset{Q1}{\underline{4}} \cdot \underset{Q2}{\underline{4}} \cdot \underset{Q3}{\underline{4}} = 64$$

b) If you guess randomly on each of these questions, what is the probability of answering all these correctly?

$$\frac{1}{64} \text{ or } \left(\frac{1}{4}\right)^3$$

The previous problem was an example of strings with replacement – symbols can be used over and over again.

Theorem (Strings with Replacement)

Let S be a set with n elements. Then there are n^k possible strings with replacement of length k with elements from S.

Example 3: Some states allow license plates with 3 letters followed by 4 digits from 0 through 9. How many license plates are possible?

$$\underbrace{\underline{26} \cdot \underline{26} \cdot \underline{26}}_{\text{letters A-Z}} \cdot \underbrace{\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}}_{\text{digits 0-9}}$$

$$26^3 \cdot 10^4 = \boxed{175,760,000}$$

Selections with replacement are **independent events**.

For example: In the previous problem the second letter did not depend upon what the first letter was.

Independent event problems use the Multiplication Counting Principle to find probability.

Definition of Independent Events

Events A and B are **independent events** if and only if $P(A \cap B) = P(A) \cdot P(B)$.

Events that are not independent are called **dependent events**.

Example 4: In a certain spinning wheel, there are 20 sectors of equal size. In 18 of these sectors, you win a prize, but in the other 2, you lose all your winnings. If the wheel spins randomly, what is the probability of winning five prizes and then losing on the sixth spin?

Spins $\underset{\text{win}}{\underline{\frac{18}{20}}} \cdot \underset{\text{win}}{\underline{\frac{18}{20}}} \cdot \underset{\text{win}}{\underline{\frac{18}{20}}} \cdot \underset{\text{win}}{\underline{\frac{18}{20}}} \cdot \underset{\text{win}}{\underline{\frac{18}{20}}} \cdot \underset{\text{lose}}{\underline{\frac{2}{20}}}$

$$\frac{18^5 \cdot 2}{20^6} = .059049 = 6\%$$